

## Calculations Policy for Foundation, Key Stage 1 and Key Stage 2

Maltby Redwood Academy
Every Child an Achiever


Maltby Learning Trust

## Foundation Stage




Understanding of counting on with a numbertrack.

Understanding of counting on with a numberline (supported by models and images).

7+4


Use numicon to recall number bonds up to 20 .


Introduce bar method for addition (see bar model)
Missing number problems e.g $14+5=10+\square \quad 32+\square+$

$$
\square=100 \quad 35=1+\square+5
$$

Teach partitioning of 2 digit numbers and using dienes to represent this. Move onto drawing the dienes to represent the values.

Partitioning in different ways and recombine 47+25

Leading to exchanging:
Year 3
Missing number problems using a range of equations as in Year 1 and 2 but with appropriate, larger numbers.

## Partition into tens and ones

## Partition both numbers and recombine.

Count on by partitioning the second number only e.g.
$247+125=247+100+20+5$
$=347+20+5$
$=367+5$
$=372$
Children need to be secure adding multiples of 100 and 10 to any three-digit number including those that are not multiples of 10 .

## Towards a Written Method

Introduce expanded column addition modelled with place value counters (Dienes could be used for those who need a less abstract representation)


Leading to children understanding the exchange between tens and ones.


Some children may begin to use a formal columnar algorithm, initially introduced alongside the expanded method. The formal method should be seen as a more streamlined version of the 247 expanded method, not a new method.
$+125$

| Year 4 | Year 5 | Year 6 |
| :---: | :---: | :---: |
| Missing number/digit problems: <br> Mental methods should continue to develop, supported by a range of models and images, including the number line. The bar model should continue to be used to help with problem solving. <br> Written methods (progressing to 4-digits) <br> Expanded column addition modelled with place value counters, progressing to calculations with 4-digit numbers. $\begin{gathered} 200+40+7 \\ \frac{100+20+5}{300+60+12}=372 \\ 247 \\ +\frac{125}{12} \\ 60 \\ \frac{300}{372} \end{gathered}$ <br> Compact written method <br> Extend to numbers with at least four digits. <br> Children should be able to make the choice of reverting to expanded methods if experiencing any difficulty. <br> Extend to up to two places of decimals (same number of decimals places) and adding several numbers (with different numbers of digits). $\begin{array}{r} 72.8 \\ +54.6 \\ \hline 127.4 \\ \hline 11 \end{array}$ | Missing number/digit problems: <br> Mental methods should continue to develop, supported by a range of models and images, including the number line. The bar model should continue to be used to help with problem solving. Children should practise with increasingly large numbers to aid fluency e.g. $12462+2300=14762$ <br> Written methods (progressing to more than 4digits) <br> As year 4, progressing when understanding of the expanded method is secure, children will move on to the formal columnar method for whole numbers and decimal numbers as an efficient written algorithm. <br> Place value counters can be used alongside the columnar method to develop understanding of addition with decimal numbers. | Missing number/digit problems: <br> Mental methods should continue to develop, supported by a range of models and images, including the number line. The bar model should continue to be used to help with problem solving. <br> Written methods <br> As year 5, progressing to larger numbers, aiming for both conceptual understanding and procedural fluency with columnar method to be secured. <br> Continue calculating with decimals, including those with different numbers of decimal places <br> Problem Solving <br> Teachers should ensure that pupils have the opportunity to apply their knowledge in a variety of contexts and problems (exploring cross curricular links) to deepen their understanding. |

## Subtraction

## Foundation Stage



Missing number problems e.g. $52-8=\square$; $\square-20=25$; $22=$ $\square-21 ; 6+\square+3=11$

Recording addition and subtraction in expanded columns can support understanding of the quantity aspect of place value and prepare for efficient written methods with larger numbers. The numbers may be represented with Dienes apparatus. E.g. 75-42


It is valuable to use a range of representations (also see $\mathrm{Y} 1)$. Continue to use number lines to model take-away and difference. E.g.


The link between the two may be supported by an image like this, with 47 being taken away from 72, leaving the difference, which is 25 .


Use manipulatives alongside this blank number line method. The bar model should continue to be used, as well as images in the context of measures.


Missing number problems e.g. $\quad=43-27$; 145 = 138; $274-30=\square ; 245-\square=195 ; 532-200=\square$; $364-153=\square$
Mental methods should continue to develop, supported by a range of models and images, including the number line. The bar model should continue to be used to help with problem solving (see Y1 and Y2).
Children should make choices about whether to use complementary addition or counting back, depending on the numbers involved.

## Written methods (progressing to 3-digits)

Introduce expanded column subtraction with no decomposition, modelled with place value counters (Dienes could be used for those who need a less abstract representation)


For some children this will lead to exchanging, modelled using place value counters (or Dienes).


A number line and expanded column method may be compared next to each other.

Some children may begin to use a formal columnar algorithm, initially introduced alongside the expanded method. The formal method should be seen as a more streamlined version of the expanded method, not a new method.


## Multiplication

Foundation Stage


Understand multiplication is related to doubling and combing groups of the same size (repeated addition)

Washing line, and other practical resources for counting. Concrete objects. Numicon; bundles of straws, bead strings


Problem solving with concrete objects (including money and measures

Use cuissenaire and bar method to develop the vocabulary relating to 'times' -
Pick up five, 4 times
Use arrays to understand multiplication can be done in any order (commutative)
$0000^{4 \times 2=8}$
$0000^{4 \times 2=8}$
$2 \times 4=8$


Expressing multiplication as a number sentence using $x$ Using understanding of the inverse and practical resources to solve missing number problems.
$7 \times 2=\square$
$\square=2 \times 7$
$7 \times \square=14$

- 14 -
$\square x 2=14$
$14=2 x$
(X) $=14$
$14=\square \times$

Develop understanding of multiplication using array and number lines (see Year 1). Include multiplications not in the 2,5 or 10 times tables.

Begin to develop understanding of multiplication as scaling (3 times bigger/taller)

$4 \times 3=12$


## Towards written methods

Use jottings to develop an understanding of doubling two digit numbers.


Missing number problems
Continue with a range of equations as in Year 2 but with appropriate numbers.

## Mental methods

Doubling 2 digit numbers using partitioning
Demonstrating multiplication on a number line jumping in larger groups of amounts
$13 \times 4=10$ groups $4=3$ groups of 4

## Written methods (progressing to 2d x 1d)

Developing written methods using understanding of visual images


Develop onto the grid method

|  | 10 | 8 |
| :---: | :---: | :---: |
| 3 | 30 | 24 |
|  |  |  |

Give children opportunities for children to explore this and deepen understanding using Dienes apparatus and place value counters.

Some children may progress onto the formal method for multiplication.

| Year 4 | Year 5 | Year 6 |
| :---: | :---: | :---: |
| Continue with a range of equations as in Year 2 but with appropriate numbers. Also include equations with missing digits $\square 2 \times 5=160$ <br> Mental methods <br> Counting in multiples of $6,7,9,25$ and 1000, and steps of $1 / 100$. <br> Solving practical problems where children need to scale up. Relate to known number facts. (e.g. how tall would a 25 cm sunflower be if it grew 6 times taller?) <br> Written methods (progressing to 3d x 2d) Children to embed and deepen their understanding of the grid method to multiply up 2d x 2d. Ensure this is still linked back to their understanding of arrays and place value counters. <br> Some children may progress onto the formal method for multiplication. <br> 11 | Continue with a range of equations as in Year 2 but with appropriate numbers. Also include equations with missing digits <br> Mental methods <br> $X$ by 10, 100, 1000 using moving digits ITP <br> Use practical resources and jottings to explore equivalent statements (e.g. $4 \times 35=2 \times 2 \times 35$ ) <br> Recall of prime numbers up 19 and identify prime numbers up to 100 (with reasoning) <br> Solving practical problems where children need to scale up. Relate to known number facts. <br> Identify factor pairs for numbers <br> Written methods (progressing to 4d $\times 2 d$ ) <br> Long multiplication using place value counters <br> Children to explore how the grid method supports an understanding of long multiplication (for $2 \mathrm{~d} \times 2 \mathrm{~d}$ ) | Continue with a range of equations as in Year 2 but with appropriate numbers. Also include equations with missing digits <br> Mental methods <br> Identifying common factors and multiples of given numbers <br> Solving practical problems where children need to scale up. Relate to known number facts. <br> Written methods <br> Continue to refine and deepen understanding of written methods including fluency for using long multiplication <br> Children to use methods to multiply decimals. |

## Division

## Foundation Stage

| GUIDANCE / MODELS AND IMAGES | KEY VOCABULARY |
| :---: | :---: |
| The ELG states that children solve problems, including doubling, halving and sharing. <br> Children need to see and hear representations of division as both grouping and sharing. <br> Division can be introduced through halving. <br> Children begin with mostly pictorial representations linked to real life contexts: <br> Grouping model <br> Mum has 6 socks. She grouped them into pairs - how many pairs did she make? <br> Sharing model <br> I have 10 sweets. I want to share them with my friend. How many will we have each? <br> Children have a go at recording the calculation that has been carried out. | halve <br> share, share equally <br> qne each, two each, three each... <br> group in pairs, threes... <br> tens <br> equal groups of <br> divide <br> divided by <br> divided into <br> left, left over |

## FRACTIONS

| GUIDANCE / MODELS AND IMAGES | KEY VOCABULARY |
| :---: | :---: |
| Although not explicit in the Development Matters document, the sharing model is a useful way of introducing young children to fractions and calculating with fractions. | As division vocabulary plus: fraction |
| Setting the problems in real life context and solving them with concrete apparatus will support children's understanding. | half <br> halves |
| "I have got 5 bones to share between my two dogs. How many bones will they get each?" | third |
| Children have a go at recording the calculation that has been carried out. $21 / 2+21 / 2=5$ | thirds |


| Year 1 | Year 2 | Year 3 |
| :---: | :---: | :---: |
| Children must have secure counting skills- being able to confidently count in 2 s , 5 s and 10 s . Children should be given opportunities to reason about what they notice in number patterns. <br> Group AND share small quantitiesunderstanding the difference between the two concepts. <br> Sharing <br> Develops importance of one-to-one correspondence. $15 * 5=3$ <br> 15 shared between 5 <br> Children should be taught to share using concrete apparatus. <br> Grouping <br> Children should apply their counting skills to develop some understanding of grouping. <br> Use of arrays as a pictorial representation for division. $15 \div 3=5$ There are 5 groups of 3 . $15 \div 5=3$ There are 3 groups of 5 . <br> Children should be able to find $1 / 2$ and $1 / 4$ and simple fractions of objects, numbers and quantities. | $\begin{array}{ll} \div=\text { signs and missing numbers } \\ \hline 6 \div 2=\square & \square=6 \div 2 \\ 6 \div \square=3 & 3=6 \div \square \\ \square \div 2=3 & 3=\square \div 2 \\ \square \div \nabla=3 & 3=\square \div \nabla \end{array}$ <br> Know and understand sharing and grouping-introducing children to the $\div$ sign. <br> Children should continue to use grouping and sharing for division using practical apparatus, arrays and pictorial representations. <br> Grouping using a numberline <br> Group from zero in jumps of the divisor to find our 'how many groups of 3 are there in 15?'. $15 \div 3=5$ <br> Continue work on arrays. Support children to understand how multiplication and division are inverse. Look at an array - what do you see? | $\div=$ signs and missing numbers <br> Continue using a range of equations as in year 2 but with appropriate numbers. <br> Grouping <br> How many 6's are in 30 ? <br> $30 \div 6$ can be modelled as: <br> Becoming more efficient using a numberline <br> Children need to be able to partition the dividend in different ways. <br> Remainders <br> Sharing - 49 shared between 4 . How many left over? <br> Grouping - How many 4s make 49. How many are left over? <br> Place value counters can be used to support children apply their knowledge of grouping. <br> For example: <br> $60 \div 10=$ How many groups of 10 in 60 ? <br> $600 \div 100=$ How many groups of 100 in 600 ? |


| Year 4 | Year 5 | Year 6 |
| :---: | :---: | :---: |
| $\div=$ signs and missing numbers <br> Continue using a range of equations as in year 3 but with appropriate numbers. <br> Sharing, Grouping and using a number line <br> Children will continue to explore division as sharing and grouping, and to represent calculations on a number line until they have a secure understanding. Children should progress in their use of written division calculations: <br> -Using tables facts with which they are fluent <br> - Experiencing a logical progression in the numbers they use, for example: <br> Formal Written Methods <br> Formal short division should only be introduced once children have a good understanding of division, its links with multiplication and the idea of 'chunking up' to find a target number (see use of number lines above) <br> Short division to be modelled for understanding using place value counters as shown below. Calculations with 2 and 3 -digit dividends. E.g. fig 1 | Formal Written Methods <br> Continued as shown in Year 4, leading to the efficient use of a formal method. The language of grouping to be used (see link from fig. 1 in Year 4) <br> E.g. $1435 \div 6$ <br> Children begin to practically develop their understanding of how express the remainder as a decimal or a fraction. Ensure practical understanding allows children to work through this (e.g. what could I do with this remaining 1? How could I share this between 6 as well?) $362 \div 7=$ | $\div=$ signs and missing numbers <br> Continue using a range of equations but with appropriate numbers <br> Sharing and Grouping and using a number line <br> Children will continue to explore division as sharing and grouping, and to represent calculations on a number line as appropriate. <br> Quotients should be expressed as decimals and fractions <br> Formal Written Methods - long and short division <br> Short method $\begin{gathered} 547 \div 23= \\ 2 3 \longdiv { 5 4 ^ { 8 } 7 } \end{gathered}$ <br> Long Method |

## Progression of Bar Modelling

Sam had 10 red marbles and 12 blue marbles. How many marbles did he have altogether?


$$
10+12=22
$$

In problems involving addition and subtraction there are three possible unknowns as illustrated below and given the value of two of them the third can be found.


The examples below illustrate a variety of ways that the bar might be used for addition and subtraction problems. A question mark is used to indicate the part that is unknown.

Addition
Aggregation

- two quantities combined

Addition
Augmentation

- a quantity is increased


I have 6 red pencils and 4 yellow pencils. How many pencils do I have?
(I combine two quantities to form the whole)


I have 6 red pencils and I buy 4 yellow pencils. How many pencils do I have?
(The bar I started with increases in length)

Subtraction

- Comparison or Difference
- Take Away


I had 10 pencils and I gave 6 away, how many do I have now?
(This time we know the whole but only one of the parts, so the whole is partitioned and one of the parts removed to identify the missing part)

## Equivalence

The model can be rearranged to demonstrate equivalence in a traditional layout


Pupils need to develop fluency in using this structure to represent addition and subtraction problems in a variety of contexts using the bar model. The model will help children to see that different problems share the same mathematical structure and can be visualised in the same way. Asking children to write their own problems, using the bar as the structure will help to consolidate this understanding.

Milking the maths: using the bar model flexibly across all year groups:


What could this bar model be showing?

Can you design questions that fit the bar model?


Playing with algebra



## National Curriculum Objectives: Multiplication and Division

Year 1: solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.
Focus on verbalising thinking:
e.g.
'eight is two taken four times'
'two taken four times is eight'
'eight equals four times two'
'there are four twos in eight'


Focus on verbalising thinking:
'eight is two taken four times' 'two taken four times is eight' 'eight equals four times two' 'there are four twos in eight'

$\qquad$

Year 2: solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.
[Photograph of Cuisenaire being used as a bar model and arrays.]

## National Curriculum Objectives: Fractions

Year 1: recognise, find and name a half as one of two equal parts of an object, shape or quantity recognise, find and name a quarter as one of four equal parts of an object, shape or quantity.

Year 2: recognise, find, name and write fractions $1 / 3,1 / 4,2 / 4$ and $3 / 4$ of a length, shape, set of objects or quantity write simple fractions e.g. $1 / 2$ of $6=3$ and recognise the equivalence of $2 / 4$ and $1 / 2$. e.g.

What fractions can you see?
What fraction of the orange is each yellow piece?
If the value of the orange rod is ten, what is the value of each yellow rod?
Extension: What if the value of the orange rod is 6 ? 100 ? Etc.


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National Curriculum Objectives: Problem Solving
Year 1: solve one-step problems that involve addition and subtraction, using concrete objects and pictorial
representations, and missing number problems such as 7=ם-9.
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Year 2: solve problems with addition and subtraction:
- using concrete objects and pictorial representations, including those involving numbers, quantities and measures
- applying their increasing knowledge of mental and written methods
e.g.
I think of a number. I subtract 5 . The answer is 4 . What is my number?
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e.g.
A tub contains 24 coins. Saj takes 5 coins. Joss takes 10 coins. How many coins are left in the tub?
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Children should now be ready to start drawing bar model representations. For the purpose of this progression document, we have modelled 'before' and 'after' representations of problems. It may, however, be that schools instead choose to use a model whereby the first representation is developed.
e.g.

Aiden has seven marbles and Harvey has fifteen. They decide to share them equally between them. How many do they get each?


## National Curriculum Objectives: Problem Solving in Addition and Subtraction

Year 3: solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction.

Year 4: solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why.

- e.g. one-step problem
- Sally has 40 football cards. She gives 25 of them away. How many does she give away?


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National Curriculum Objectives: Problem Solving in Multiplication and Division
Year 3: solve problems, including missing number problems, involving multiplication and division, including integer
scaling problems and correspondence problems in which n objects are connected to m objects.
This is an example of integer scaling.
Peter has 4 books
Harry has five times as many books as Peter.
How many books has Harry?
4
44
4\times5=20
Harry has 20 books
Year 4: solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as \(n\) objects are connected to m objects.
e.g. 8 children each download 59 songs to play on their iPod. How many songs do they have altogether?
```


## National Curriculum Objectives: Fractions

Year 3: solve problems that involve (unit fractions and fractions with the same denominator)
Year 4: solve problems involving increasingly harder fractions to calculate quantities, and fractions to divide quantities,
including non-unit fractions where the answer is a whole number
Year 4: solve simple measure and money problems involving fractions and decimals to two decimal places, including decimals e.g. A computer game is $£ 24$ in the sale. This is one quarter of its original price. How much did it cost before the sale?


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Cuisenaire and double-sided counters are still valuable resources to use alongside drawn representations of problems.
They should all be used flexibly according to the stage that children have reached in their learning.
National Curriculum Objectives: Problem Solving in Addition and Subtraction
Year 5: solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use
and why
Year 6: solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use
and why
National Curriculum Objectives: Problem Solving in Multiplication and Division
Year 5: solve problems involving multiplication and division, including scaling by simple fractions and problems involving
simple rates
e.g. How many jugs with a capacity of 250ml could you fill with 10 litres of water?
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## Example Questions for Key Stage 1

## Year 1 Problems

1. Ebony has 5 p and Daniel has 8 p. How much do they have altogether?
2. A lolly costs 6 p. Amrit paid with a 10p coin. How much change does he get?
3. Michael says that $16+5=21$. Is he correct?
4. I think of a number. I subtract 5 . The answer is 4 . What was my number?
5. How many gloves are there altogether in 6 pairs of gloves?
6. Twelve people are split into two groups. How any are in each group?
7. Mrs Morton puts five 5 p coins into her purse. How much is in her purse altogether?

## Year 2 Problems

1. Dylan has 37 coloured pencils and he buys 30 more. How many does he have now?
2. Janie has 40 beads. She loses 25 of them. How many does she have left?
3. What is the difference between seventy six and thirty five?
4. I think of a number. I subtract 5 . The answer is 4 . What was my number?
5. Last week Ellie got $£ 1.00$ pocket money. She spent half of it. How much has she got left?
6. A tub contains 24 coins. Saj takes 5 coins. Joss takes 10 coins. How many coins are left in the tub?
7. Amelia writes the calculation below as a multiplication calculation? What might she write?
$3+3+3+3+3=15$
8. Mr Siddique shares $£ 18$ equally between his three sons. How much does each son get?
9. Charlotte-May had to find a 14 of a number. Her answer was 4 . What number did she start with?
10. Danny cuts his pizza into 8 equal slices. He eats $3 / 4$ of the pizza and gives the rest to his dog, Gruff. How many pieces does Danny eat?

## Example Questions for Lower Key Stage 2

## Year 3 Problems

1. There are 334 children at Springfield School and 75 at Holy Trinity Nursery. How many children are there altogether?
2. Gemma collected 293 badges but she gave 45 of them to her friend, Rebecca. How many badges did she have left?
3. Aiden has seven marbles and Harvey has fifteen. They decide to share them equally between them. How many do they get each?
4. Seven people each put five pens into a pot. Carmen then takes out fifteen pens. How many pens are left?
5. If you spend 61 p at the corner shop, how much change do you get from $£ 1.00$ ?
6. If five apples cost fifty pence, how much would two apples cost?
7. Emma buys seven markers for 30 p each. How much change does she get from $£ 3.00$ ?
8. A bookcase in the library holds 5 shelves with 46 books on each shelf. How many books are there in the bookcase altogether?

9 . How many 5 p stickers can Alexis buy with his 55 p pocket money?
10. Which is the larger amount, one third of $£ 60$ or one quarter of $£ 88$ ?
11. A computer game is $£ 24$ in the sale. This is one quarter of its original price. How much did it cost before the sale?

## Year 4 Problems

1. Martin has saved $£ 6.78$ and spends $£ 4.69$. How much does he have left?
2. Sally has 40 football cards. She gives 2 fifths of them away. How many does she give away?
3. Sally has 30 football cards. She gives 2 fifths of them to her friend. How many does she have left?
4. 8 children each download 59 songs to play on their iPod. How many songs do they have altogether?
5. Calculate how many fives there are in 85 ?
6. At the dressmakers, Debbie buys buttons weighing 3 grams each. If she has 81 grams of buttons, how many buttons does she buy?
7. Kelly buys four fifths of the shop's oranges. If the shop had 20 oranges, how many does she have?

## Example Questions for Upper Key Stage 2

## Year 5 Problems

- 1. Every day for 4 days Helen scored 7.5 in a test. On the fifth day she scored 8 . What was her total score?
- 2 . I cut 60 cm from 3.3 m of string and shared the rest between 3 friends. How much string did they get each?
- 3. How many jugs with a capacity of 250 ml could you fill with 10 litres of water?
- 4. All the children in the school are going on a residential trip to the outdoor activity centre. They will be divided into 6 equal groups If there are 246 children in the school how many will be in each group?
- 5. Robert calculated $25 \%$ of 600 . What answer does he get?
- 6. Sam calculated $40 \%$ of 120 . What answer does he get?
- 7. Rita worked out that one sixth of a number was 12 . What was the number she started with?


## Year 6 Problems

- 1. Three quarters of a number is 54 . What is the number?
- 2. Which is more; five ninths of 252 or four sevenths of 238 ?
- 3. There are 36 packets of biscuits. One half are chocolate, a ninth are digestive and a third are wafer biscuits. The rest are ginger nuts. How many biscuits are ginger nuts?
- 4. There is $20 \%$ off in a sale. How much would a track suit cost, if the normal price was $£ 44.50$ ?
- 5. There is $20 \%$ off in a sale. The reduced price of the jeans is $£ 36$. What was the original price?
- 6. At a dance there are 4 girls to every 3 boys. There are 63 children altogether? How many girls are there?
- 7. Seven in every nine packets of crisps in a box are salt and vinegar. The rest are plain. There are 63 packets of salt and vinegar crisps. How many packets of plain crisps are there?


## Key Stage 3 Problems

1. Ralph posts 40 letters, some of which are first class, and some of which are second class.

He posts four times as many second class letters as first.
How many of each class of letter does he post? (This question appeared on a GCSE higher tier paper.)
2. A computer game was reduced in a sale by $20 \%$ and it now costs $£ 55$. What was the original cost?
3. A computer game was reduced in a sale by $30 \%$ and it now costs $£ 77$. What was the original cost?

